CAPRA COACHING

Year 10 Exam Booklet:
Algebra and Equations

Algebra and Equations **Name: ……**

Easy

1. Express $\frac{1}{3x-2} - \frac{1}{3x+2}$ as a single fraction.

2. Solve each quadratic equation using the method specified:

   i. $x^2 + 3x + 1 = 0$ by the quadratic formula,

   ii. $x^2 - 2x - 1 = 0$ by completing the square.

Algebra and Equations **Name: ……**

3. Solve each of the following quadratic equation by the factorizing:

i. $x^2 + 4x = 0$

ii. $x^2 + 11x - 26 = 0$

iii. $2x^2 - x - 6 = 0$

4. Simplify:

$$\frac{6}{9 - x^2} - \frac{2}{3 - x}$$

Algebra and Equations **Name: ……**

5. Make r the subject in this equation:

$$x = \frac{3r}{r+2}$$

6. Solve $4v^2 + 5v - 6 = 0$

7. Solve each equation to find the value of x:

i. $\frac{3}{x} = \frac{4}{5}$

ii. $3x - 5 = \frac{5+x}{2}$

iii. $(3x - 5 = \frac{5+x}{2}$

8. Solve these equations simultaneously for x and y:
$5x - 2y = -16$
$3x + 4y = -7$

Algebra and Equations **Name: ……**

9. Solve $2(3-x) \leq 4-x$ and graph of the answer on a number line.

10. How many solutions does the quadratic equations $2x^2 - 2x - 3$ have?

(A). 0
(B). 1
(C). 2
(D). 3

11. Find the monic quadratic equation that has solutions $x = -4$ and $x = 7$. Give your answer in expanded form.

Algebra and Equations **Name: ……**

12. If the equation $S = \frac{a}{1-r}$ is rearranged to make r the subject of the formula, then the answer is:

(A). $r = 1 - \frac{a}{s}$
(B). $r = \frac{a-s}{s}$
(C). $r = \frac{a}{s} - 1$
(D). $r = \frac{1-s}{a}$

13. Solve the following simultaneous equations:

$$y = x - 2$$
$$2x + y = 7$$

14. Consider the quadratic $ax^2 + bc + c = 0$. If the value of b^2-4ac is less than zero, then the equation will have how many solutions?

(A). None
(B). One
(C). Two
(D). Many

Algebra and Equations **Name: ……**

15. A quadratic equation was solved to find that solutions were $x = 4$ and $x = -4$. What was the original equation?

(A). $x^2 + x - 12 = 0$
(B). $x^2 - x - 12 = 0$
(C). $x^2 - x - 1 = 0$
(D). $2x - 1 = 0$

16. What is the correct solution to the equation $9x^2 - 4 = 0$?

(A). $\pm\frac{2}{3}$
(B). $\pm\frac{3}{2}$
(C). $\pm\frac{4}{9}$
(D). $\pm\frac{9}{4}$

17. Solve the following equations using the most appropriate methods
$x^2 + 2x - 15 = 0$

Algebra and Equations **Name: ……**

18. Solve $(x-3)(x+1)=0$.

19. Factories x^2-6x+9.

20. Make x the subject of the formula $A=\frac{1}{2}xy$.

Algebra and Equations **Name: ……**

21. Expand and simplify $(2x + 1)(x - 3)$.

22. Solve $x^2 - 7x + 12 = 0$.

23. Solve $\frac{x-2}{3} - \frac{x}{2} = \frac{1}{6}$.

Algebra and Equations **Name: ……**

Medium

24. Simplify $\frac{3a^2-2a-1}{3a+1} \div \frac{2a^2-1}{a+1}$.

25. Simplify $\frac{y^2-2y-3}{2y^2-y-3} \div \frac{3y-y^2}{4y-6}$.

Algebra and Equations **Name: ……**

26. Solve $\frac{2}{x}+\frac{x}{3}=\frac{11}{6}$.

27. Luca cycled 140km at a uniform speed of $x\ km/h$. If he had travelled $15km/h$ slower, the same journey would have taken an additional 3 hours.

i. Write down an expression for the time Luca takes for the journey.

ii. Form an equation and solve it to find Luca's actual speed.

Algebra and Equations **Name: ……**

28. Factorise the denominators and then simplify $\frac{8}{x^2+2x-3} - \frac{14}{2x^2+5x-3}$.

29. The width of a rectangle is $7cm$ shorter than length of the rectangle. If the area of the rectangle is $30cm$ find the length of rectangle.

Algebra and Equations **Name: ……**

30. Solve simultaneously
$y = 2x^2$
$y = 5x + 3$

31. Use the substitution $u = \sqrt{x}$ in the following equation, then solve for x.
$x - 14\sqrt{x} + 24 = 0$

Algebra and Equations **Name: ……**

32.

i. Factorise $d^2 - d - 12$

ii. Hence, simplify: $\frac{d-4}{20} \div \frac{d^2+d-12}{5d}$.

Algebra and Equations **Name: ……**

33. $3(x+2)^2 + 7(x+2) - 4$ is equivalent to:

(A). $3x^2 + 7x - 4$
(B). $3x^2 + 7x - 2$
(C). $3x^2 - 5x - 6$
(D). $3x^2 + 19x + 22$

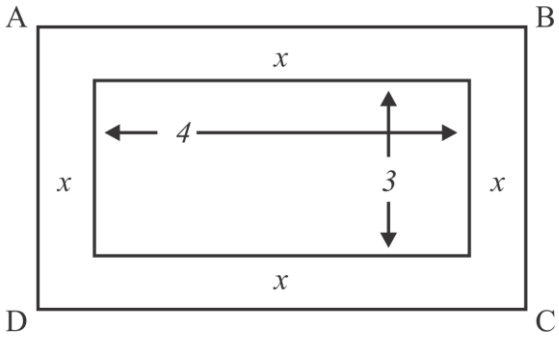
34. Factorise fully, $x^4 - 81$

35. In eight hours Jake walks twelve kilometers more than Fiona does in seven hours, and in thirteen hours Fiona walks seven kilometers more than Jake does in nine hours if Jake walks at x kilometers per hour and Fiona walks at y kilometers per hour, form a pair of simultaneous equations and solve them to find how fast each of Jake and Fiona walk.

Algebra and Equations **Name: ……**

36. The golden ratio is defined as the positive number that satisfies $a = \frac{1}{1+a}$. What is value of a?

37.



ABCD is a garden measuring 3 meters by 4 meters surrounded by a pathway that is x meters wide. If the total area of ABCD is $42m^2$:

i. Show that the area of ABCD is given by the equation $4x^2 + 14x - 30 = 0$.

ii. Hence, find the width of the pathway.

38. Alicia was solving the equation $\frac{2m-1}{4} - \frac{m-4}{3} = 6$ but made one or more mistakes. In which line of working did she make her FIRST mistake?

$\frac{2m-1}{4} - \frac{m-4}{3} = 6$

$\frac{6m-4}{12} - \frac{4m-16}{12} = 6$……Line A

$\frac{6m-3-4m-16}{12} = 6$……Line B

$2m - 17 = 72$……Line C

$2m = 55$……Line D

$m = \frac{55}{2}$

(A). Line A
(B). Line B
(C). Line C
(D). Line D

39. Solve for x: $x^4 - 7x^2 + 10 = 0$

Algebra and Equations **Name: ……**

40. Alice is answering the 20 multiple choice question on her Mathematics exm. She score 4 points for every correct answer and loses 1 point for every incorrect answer

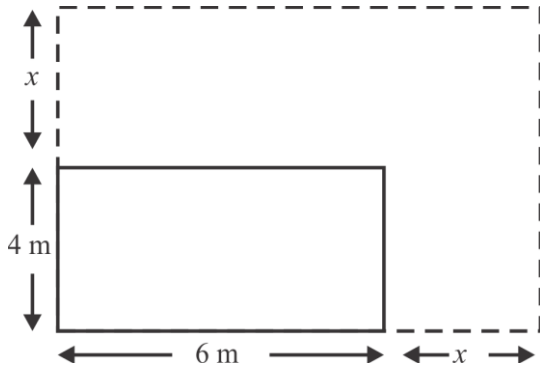
    i. If Alice scored 40 points, write a pair of simultaneous equation that can be used to represent this problem. Let x represent the number of correct answers and let y represent the number of incorrect answers.

    ii. Hence, solve your equations and calculate the number of correct and incorrect answers Alice obtained.

Algebra and Equations **Name: ……**

41. At the zoo, a rectangular wombat enclosure measuring $4m \times 6m$ is to be enlarged to accommodate some recent arrivals.

i. Write an expression for the area of the new enclosure if the same length is to be added to two side as shown in the diagram below.



ii. The new enclosure is to have double the area of the existing one. Find the dimensions of the new wombat enclosure.

Algebra and Equations **Name: ……**

42. Use the quadratic formula to solve $3x^2 + 4x - 7 = 0$.

43. Solve $m^2 - 8m - 3 = 0$ using the method of completing the square, giving the answer in exact form.

Algebra and Equations **Name: ……**

Hard

44.

i. Factorise $u^2 - 8u + 12$.

ii. Hence factorise $(x^2 - x)^2 - 8(x^2 - x) + 12$.

Algebra and Equations **Name: ……**

45.

i. Complete the square on x for $x^2 + 2bx + 2b - 1$.

ii. Using part (i) or otherwise, show that the expression has factorized form $(x + 1)(x + 2b - 1)$

iii. For what value of b will the equation $x^2 + 2bx + 2b - 1 = 0$ have only one solution?

Algebra and Equations **Name: ……**

46. One solution of the quadratic $6x^2 - 11x + n = 0$ is $x = -1\frac{2}{3}$.

47. Given that $(x + 1)(x - 1) = 5$, find the value of $(x^2 + x)(x^2 - x)$.

Algebra and Equations **Name: ……**

48.

A. The quadratic equation $x^2 + bx + c = 0$ has solutions $x = \alpha$ and $x = \beta$.
By using the quadratic formula, or otherwise, show that:

i. $\alpha + \beta = -b$

ii. $\alpha\beta = c$

B. Suppose that the quadratic equation $x^2 + kr - \frac{1}{2k^2} = 0$ (where $k \neq 0$ has solutions $x = \alpha$ and $x = \beta$.

i. Use the result in part (b) above prove that $\alpha^4 + \beta^4 = k^4 + \frac{1}{2k^4} + 2$.

ii. Hence determine the minimum possible value of $\alpha^4 + \beta^4$.

Algebra and Equations **Name: ……**

49. During a thunderstorm you see the lightning before you here the thunder. This because light travels faster than sound.

    i. Write an equation for this direct variation, using d for distance from the storm, and t for the time interval.

    ii. If the sound of thunder from lighting 2100 m away takes 6s to reach you, how far away is a storm where the time interval is 4 s?

    iii. The speed of light is approximate $3 \times 10^8 ms^2$. Therefore, we may, without loss of accuracy, assume that the light reaches you the same instant that the lighting occurs. Estimate the speed of sound.

iv. How long would it take for the sound of thunder to be head from a storm 5 km away?

50. The number of minute needed to solve an exercise set of variation problems varies directly as the number of problems and inversely as the number of people working on the solutions. It takes 4 people 36 minutes to solve 18 problems are made more difficult so that each problem takes 40% longer to solve, how many minutes will take 6 people to solve 42 problems?