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HSC Extension 1 Mathematics Exam Booklet:
Circle Geometry

Circle Geometry	Name: ………………………

Medium

1.
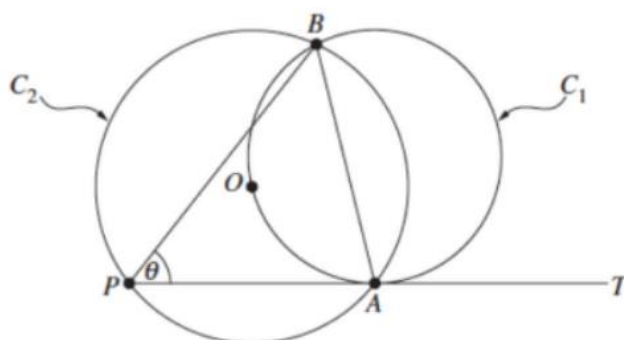
Two circles, C_1 and C_2, intersect at point A and B. Passes through the center O of circle C_2. The point P lies on circle C_2 so that the line PAT is target to circle C_1 and point A. Let $\angle APB = \theta$.

Copy or trace the diagram into your writing booklet.

i.   Find $\angle AOB$ in terms of θ. Give a reason for your answer.
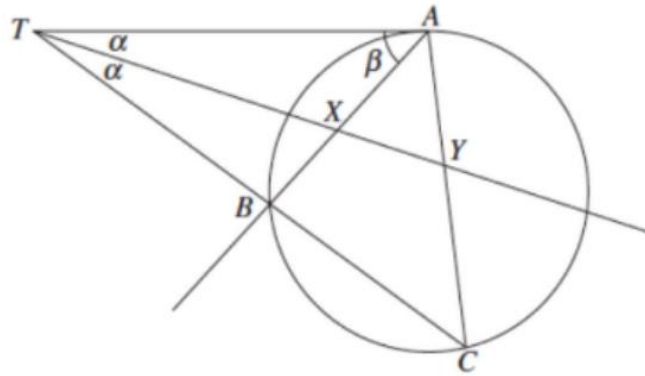
Circle Geometry   **Name: …………………………**

ii.   Explain why $\angle TAB = 2\theta$.

iii.   Deduce that $PA = BA$.

Circle Geometry   **Name:** …………………………

2.

In the diagram the point, A, B and C lie on the circle and CBproduce meets and tangent from A at
the point T. The bisector of the angle ATC intersects AB and AC at X and Y respectively.
Let ∠TAB = β.

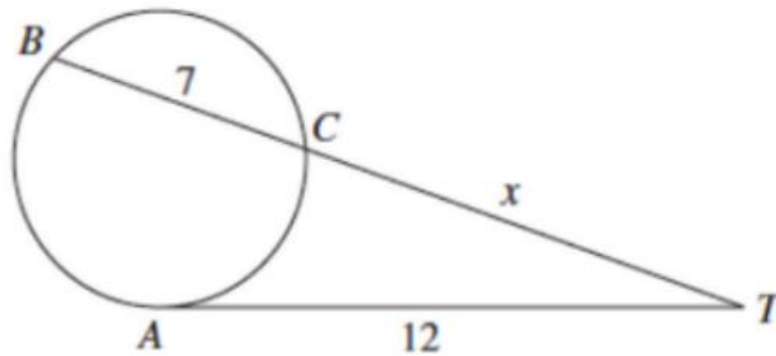Copy and trace the diagram into your writing booklet.

i.   Explain why ∠ACB = β.

ii.   Hence prove that triangle AXY is isosceles.

Circle Geometry **Name:** …………………………

3.
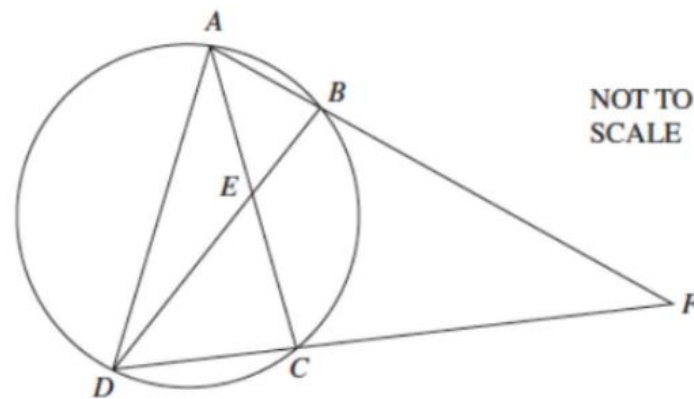

The line AT is the tangent to the circle at A, and BT is a screen meeting the circle at B and C.

Given that $AT = 12, BC = 7$ and $CT = x,$ find the value of x.

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4.

NOT TO SCALE

The point A, B, C and D are placed on a circle of radius r such that AC and BD meet at E. The line AB and Dc are produced to meet at F, and BECF is a cyclic quadrilateral. Copy or trace the diagram into your writing booklet.
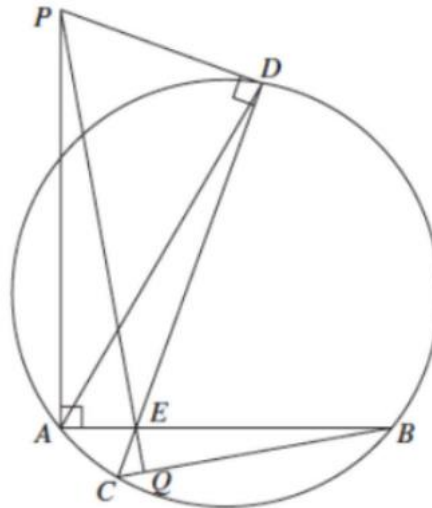
i.   Find the size of ∠DBF, giving reason for your answer.

ii.   Find an expression for the length of AD in terms of r.

Circle Geometry   Name: …………………………

5.   Two chords of a circle, AB and CD, intersect at E. The perpendicular to AB at A and CD at D intersect at P. The line PE meets BC at Q, as shown in the diagram.

i.   Explain why DPAE is a cyclic quadrilateral.

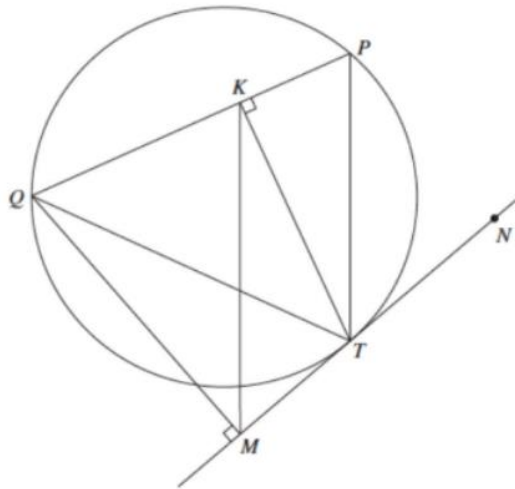**Circle Geometry**   **Name: …………………………**

ii.   Prove that ∠APE = ∠ABC.

iii.   Deduce that PQ is perpendicular to BC.

Circle Geometry   **Name: …………………………**

6.

The point P, Q and T lies on a circle. The line MN is tangent to the circle at T with M
      chosen so that QM is perpendicular to MN. The point K on PQ is chosen so that
      TK is perpendicular to PQ as shown in the diagram.

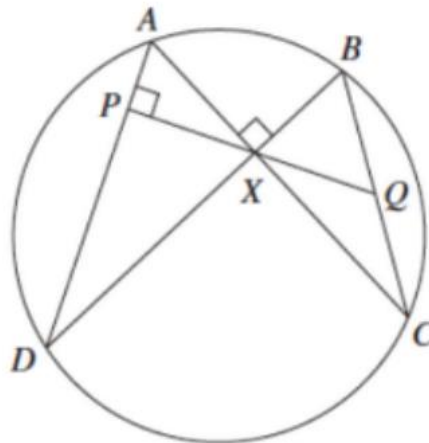i.   Show that $QKTM$ is a cyclic quadrilateral.

Circle Geometry   **Name: …………………………**

ii.   Show that $\angle KMT = \angle KQT$.

iii.   Hence, or otherwise, show that MK is parallel to TP.

Circle Geometry	Name: …………………………

7.

The diagram shows point A, B, C and D on a circle. The lines AC and BD are perpendicular and intersect at X. The perpendicular to AD through X meets AD at P and BC at Q.

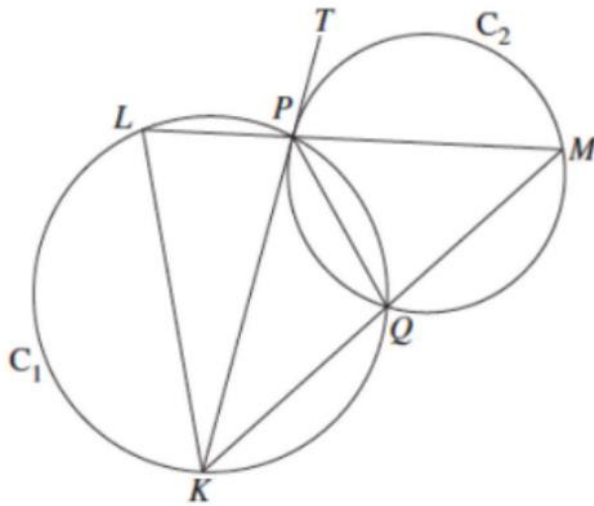Copy or trace this diagram into your writing booklet.

i.   Prove that ∠QXB = ∠QBX.

ii.   Prove that Q bisects BC.

Circle Geometry   Name: …………………………

8.

Two circles C_1 and C_2 intersect at P and Q as shown in the diagram. The tangent TP to C_2
at M. The line MP meets C_1 at L.
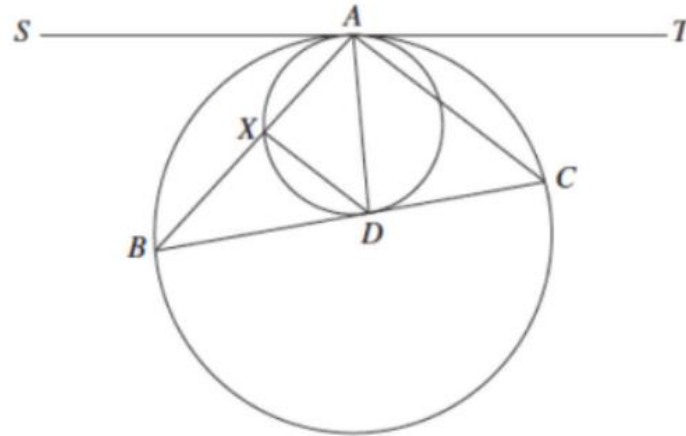Copy or trace the diagram into your writing booklet.
Prove that ΔPKL is isosceles.

Circle Geometry   **Name: ....**

9.   In the diagram, ST is tangent to both the circle at A.
        The point B and C are on the larger circle, and the line BC is tangent to the
        smaller circle at D . The line AB intersects the smaller circle at X.

Copy or trace the diagram into your writing booklet.

i.   Explain why $\angle AXD = \angle ABD + \angle XDB$.

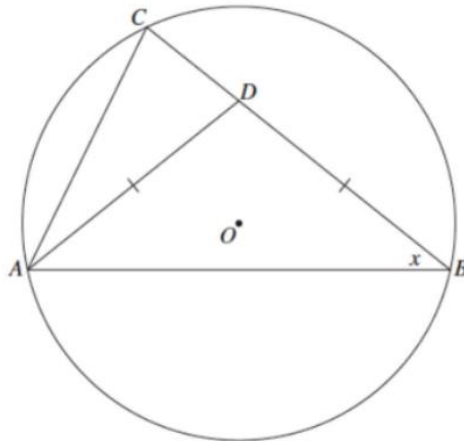**Circle Geometry**   **Name:** …………………………

ii.   Explain why $\angle AXD = \angle TAC + \angle CAD$.

iii.   Hence show that AD bisects $\angle BAC$.

Circle Geometry   **Name:** …………………………

10.   In the diagram, the vertices of ΔABC lie on the circle with center O. The point D
       lies on BC such that ΔABD is isosceles and ∠ABC = x.

       Copy or trace the diagram into your writing booklet.
i.   Explain why ∠AOC = 2x.

Circle Geometry   **Name: …………………………**
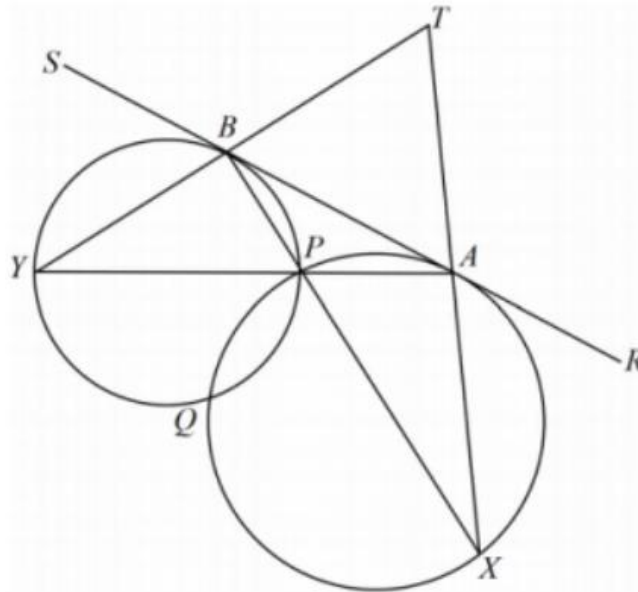
ii.   Prove that ACDO is a cyclic quadrilateral.

iii.   Let M be the midpoint of AC and P the center of the circle through A, C, D and O.
        Show that P, M and O are collinear.

11.   AB is a common tangent in two circles which intersect at P and Q as illustrated in the diagram below.
       XPB and YPA are straight lines. XA and YB intersect at T.

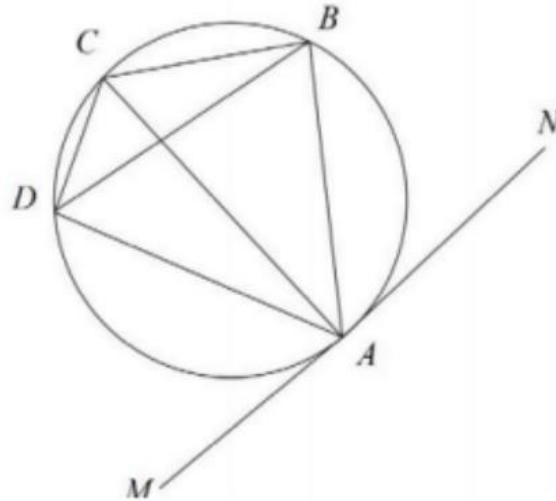i.   Copy or trace the diagram into your writing booklet.

Circle Geometry   **Name:** …………………………

ii.   Explain why ∠SBY = ∠BPY

iii.   Prove that AT = TB.

Circle Geometry   **Name: …………………………**

12.   ABCD is a cyclic quadrilateral. MAN is the tangent at A to the circle through A, B, C and D. CA bisects ∠DCB.
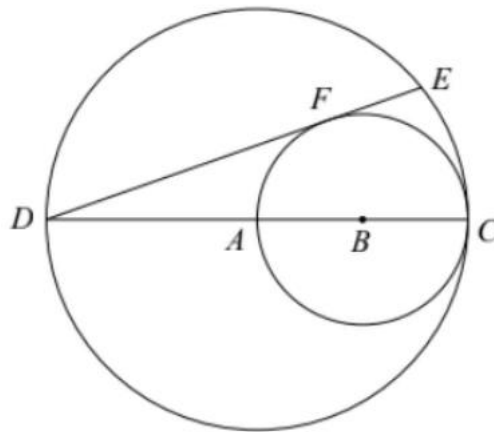

Copy or trace the diagram into your writing booklet.

i.   Explain why ∠BAN = ∠ACB.

ii.   Hence, or otherwise show that MAN||DB.

Circle Geometry   **Name:** …………………………
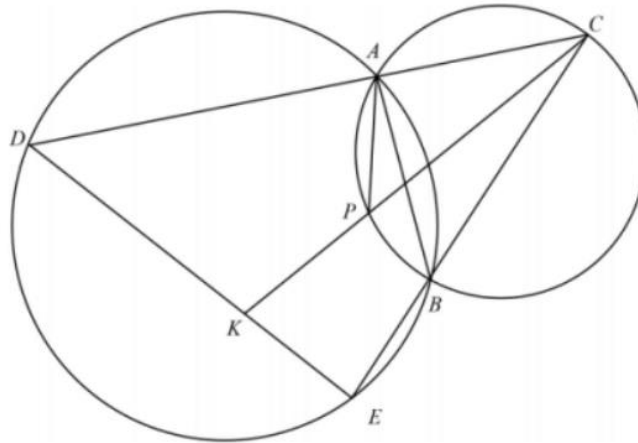
13.   In the diagram below, DC is a diameter of the larger circle centered A. AC at is a
        diameter of the smaller circle centered at B.
DE is tangent to the smaller circle at F and DC = 12.
        Copy the diagram into your answer booklet.
        Determine the length of DE.

Circle Geometry   Name: …………………………

14.

The circle intersects at A and B. The lines DAC. EBC, KPC and DKE are all straight lines.
i.   Copy or trace the diagram into your writing booklet.

Circle Geometry   **Name: …………………………**
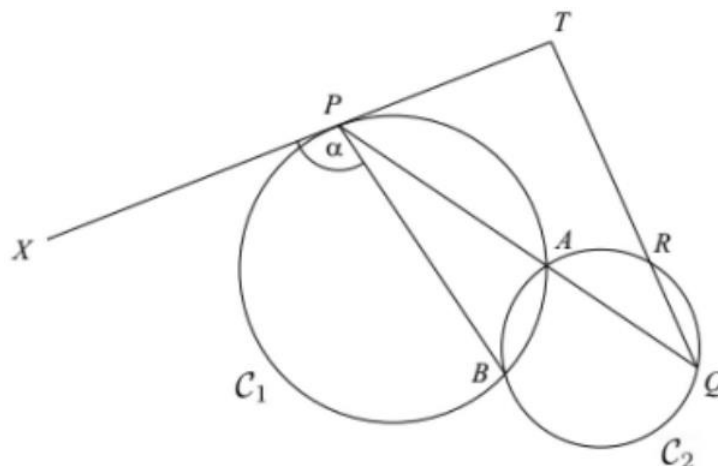
ii.   Give a reason why $\angle CBA = \angle CPA$.

iii.   Hence or otherwise, show that PADK is a cyclic quadrilateral.

Circle Geometry   **Name: …………………………**

15.

Two circles C_1 and C_2 intersect at A and B. A line through A meets the circle at P and Q respectively. A tangent is drawn from an external point T to touch the circle C_1 at P. The line TQ intersects C_2 at R.

i.   Given $\angle XPB = \alpha$, show that $\angle BRQ = 180^\circ - \alpha$, giving reasons.

ii.   Hence show that PTRB is a cyclic quadrilateral.

Hard

16.   The diagram shows a large semicircle with diameter AB and two smaller
        semicircles with diameter AC and BC , respectively, where C is appoint on the
        diameter AB . The point M is the center of the semicircle with diameter AC.
        The line perpendicular to AB through C meets the largest semicircle at the point
        D. The points S and T are the intersection of the lines AD and BD with the smaller
        semicircles. The point X is the intersection of the lines CD and ST .
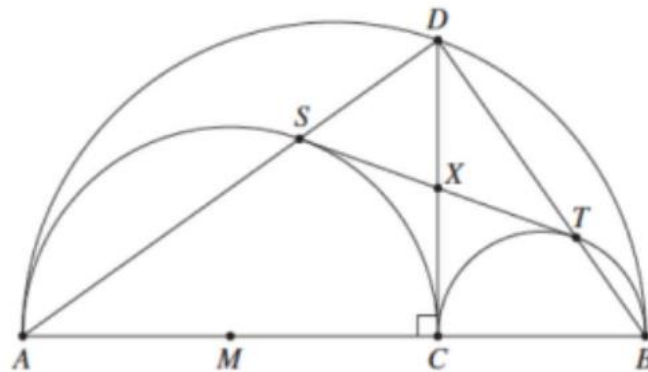
Copy or trace the diagram into your writing booklet.
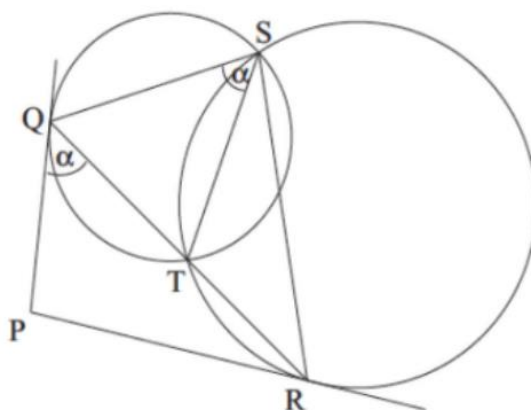
i.   Explain why CTDS is a rectangle.

ii.   Show that ΔMXS and ΔMXC are congruent.

iii.   Show that the line ST is a tangent to the semicircle with diameter AC .

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17.

In the diagram above PQ and PR are tangents to the circles SQT and STR respectively, and the points Q, T and R are collinear.

i.   Given that ∠QST = α, state a reason why ∠PQT = α.

ii.   Prove that PQSR is a cyclic quadrilateral.

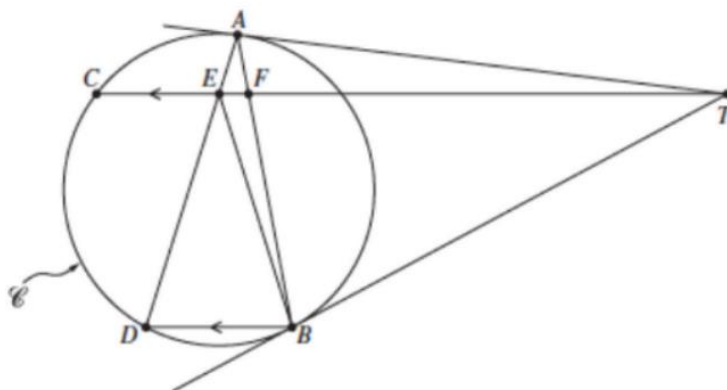**Circle Geometry**   **Name: …………………………**

18.

The diagram above shows two circles intersection at A and B.The points P, A and Q are
collinear, and the chords PM and NQ, when produced, intersect at C. Let
∠PAB = α.

i.   Give a reason why ∠BNQ = α.

ii.   Prove that the quadrilateral CMBN is cyclic.

19.

In the diagram, C is a circle with exterior point T. From T tangent are drawn to the point
A and B on C and a line TC is drawn, meeting the circle at C. The point D is the
point on C such that BD is parallel to TC. The line TC cuts the line AB at F and the
lines AD at E.
Copy or trace the diagram into your writing booklet.

i.   Prove that ΔTFA is similar to ΔTAE.

ii.   Deduce that $TE.\,TF = TB^2$.

Circle Geometry   **Name:** …………………………

iii.   Show that ΔEBT is similar to ΔBFT.

iv.   Prove that ΔDEB is isosceles.

Circle Geometry   Name: …………………………

20.
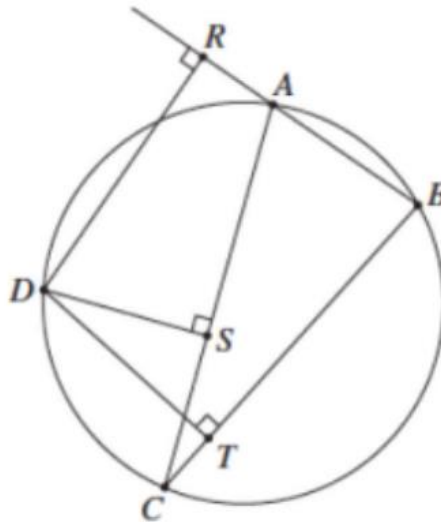
In the diagram, A, B, C and D are concyclic, and the points R, S, T are the feet of the
perpendicular from D to BA produced, AC and BC respectively.
i.   Show that ∠DSR = ∠DAR.

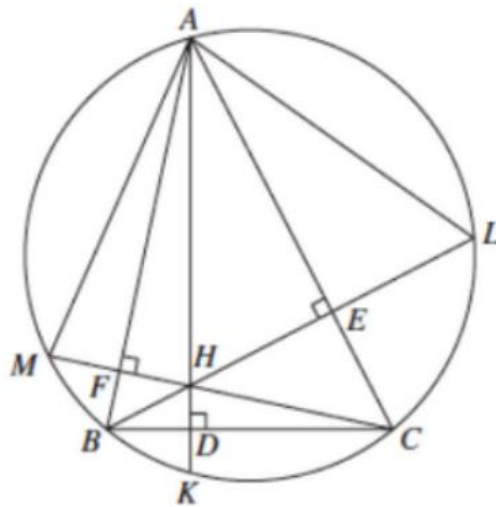**Circle Geometry**   **Name:** …………………………

ii.   Show that $\angle DST = \pi - \angle DCT$.

iii.   Deduce that the points R, S and T are collinear.

Circle Geometry   **Name: …………………………**

21.   The vertices of an acute-angles triangle ABC lie on a circle. The perpendiculars from A, B and C meet , BC, AC and AB at D, E and F respectively. These perpendiculars meet at H.
The perpendiculars AD, BE and CF are produces to meet the circle at k, L and M respectively.

i.   Prove that $\angle AHE = \angle DCE$.

ii.   Deduce that $AH = AL$.

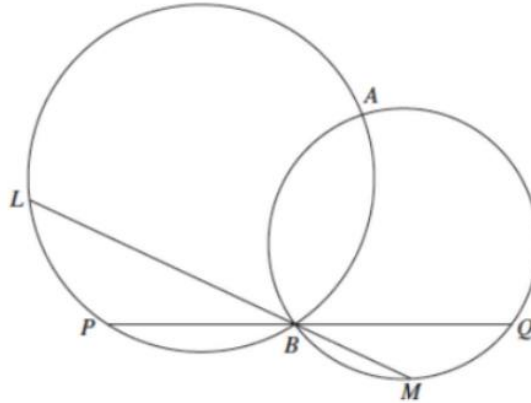**Circle Geometry**   **Name: …………………………**

iii.   State a similar result for triangle AMH.

iv.   Show that the length of the arc BKC is half the length of the arc MKL.

Circle Geometry   **Name: …………………………**

22.

Two circles intersect at A and B.

The lines LM and PQ pass through B, with L and P on one circle and and on the other circle, as shown in the diagram.

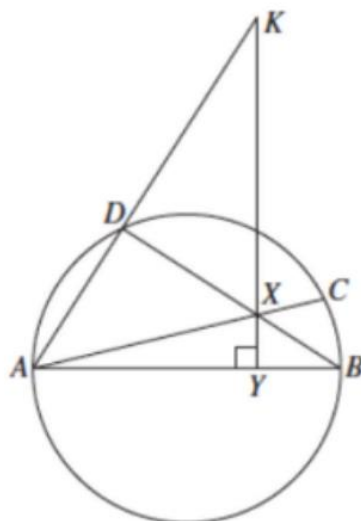Copy or trace the diagram into your writing booklet.

Show that ∠LAM = ∠PAQ.

Circle Geometry   **Name:** …………………………

23.   In the diagram AB is the diameter of the circle. The chords AC and BD intersect at X. The point Y lies on AB such that XY is perpendicular to AB . The point K is the intersection of AD produced and XY produced.

Copy or trace the diagram into your writing booklet.

i.   Show that $\angle AKY = \angle ABD$.

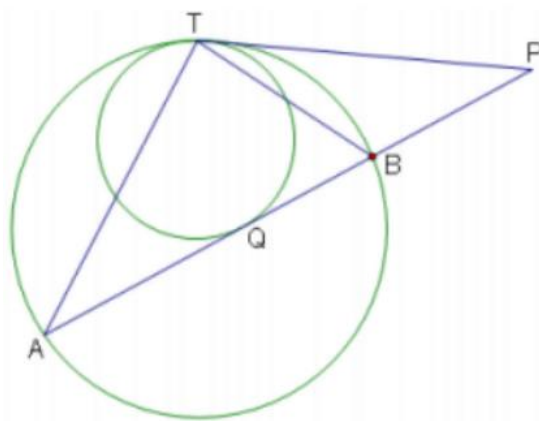**Circle Geometry**   **Name:** …………………………

ii.   Show that CKDX is a cyclic quadrilateral.

iii.   Show that B, C and K are collinear.

Circle Geometry   **Name:** …………………………

24.   PT is a common tangent to the circles with touch at T. PA is a tangent to the smaller circle at Q.

i.   Prove that ΔBTP is similar to ΔTAP.

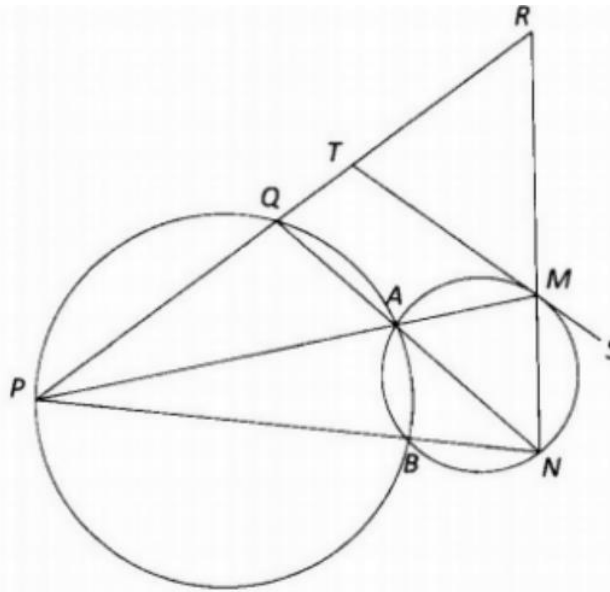**Circle Geometry**   **Name: …………………………**

ii.   Hence show that $PT^2 = PA \times PB$.

iii.   If $PT = t, QA = a$ and $QB = b$ prove that $t = \frac{ab}{a-b}$.

Circle Geometry   **Name: …………………………**

25. In the diagram, the two circles intersect at A and B. P is a point on one circle. PA and PB produced meet the other circle at M and N respectively. NA produced meets the first circle at Q. PQ
and NM produced meet at R. The tangent to the second circle meets PR at T.

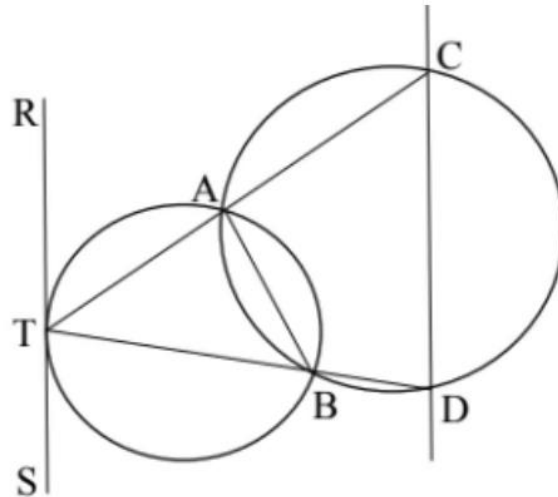

Copy or trace the diagram into your writing booklet.

i. Show that QAMR is a cyclic quadrilateral.

ii. Show that $TM = TR$.

Circle Geometry   **Name: …………………………**

26.   In the diagram, two unequal circles intersect at A and B. The line RS is tangential
        to the smaller circle at T. The lines TA and TB meet the larger circle at C and D
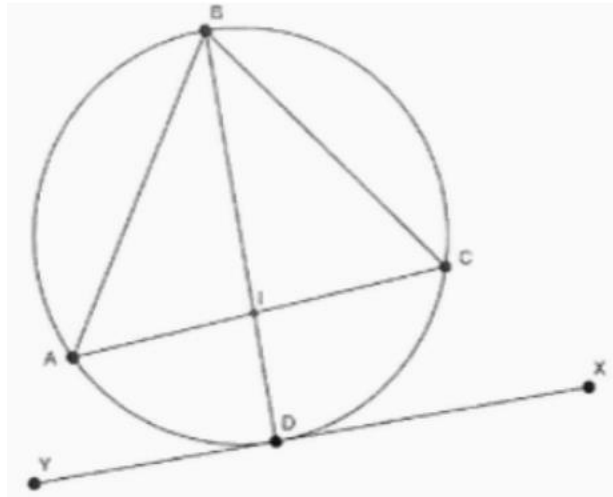   respectively.

Copy or trace the diagram into your writing booklet.

i.   State a theorem to explain why ∠BAT = ∠BDC.

ii.   Prove that RS||CD .

Circle Geometry   **Name: …………………………**

27.

In the circle, $\angle ABD + \angle BCA = 90°$, and XY is tangent to the circle at D.
The chords AC and BD intersect at l. (copy or trace the diagram in your writing booklet.)

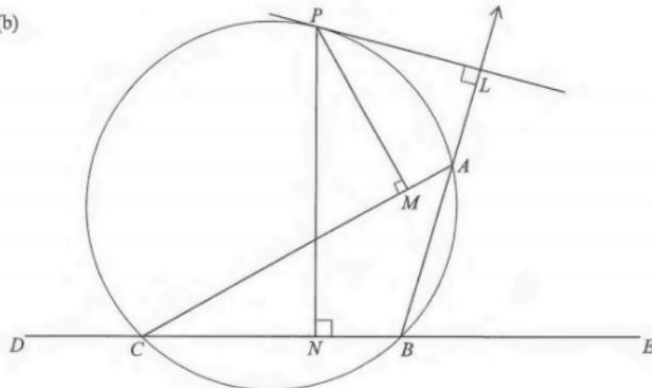i.   Prove $\angle BCD = 90°$ and hence that BD is a diameter of the circle.

ii.   Prove that if $\triangle ABC$ is isosceles with $AB = BC$, then AC||BC, then AC||XY..

28.

ABC is a triangle inscribed in a circle L, M and N are the feet of the perpendiculars from P to AB, AC and BC respectively.

i.   Copy the diagram.

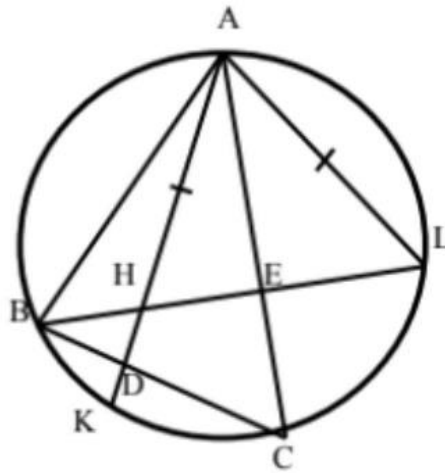ii.   Show , P, M, A and L are concyclic points.

Circle Geometry   **Name: …………………………**

iii.   Show P, C, N and M are concyclic points.

iv.   Show that L, M and N are collinear.

Circle Geometry   **Name:** …………………………

29.   Given that ALCK is a cyclic quadrilateral and H is a point on AK such that
        AH = AL. LH produced meets the circle again at B and meets AC at E. BC meets
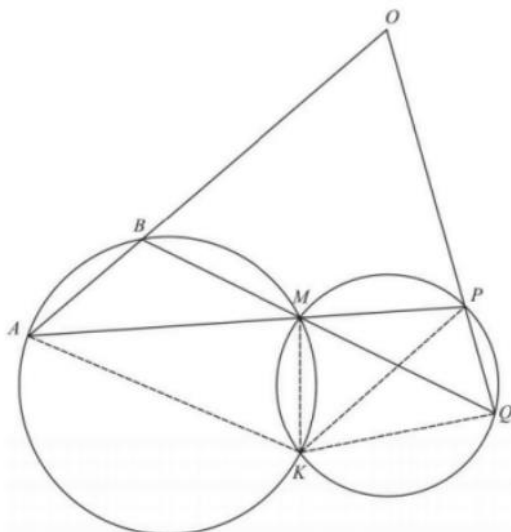   AK at D.

i.   Prove that ∠AHL = ∠ACB.

ii.   Hence state why that HECD is a cyclic quadrilateral.

iii.   Given are KC = arc CL, prove that HC is a diameter of HECD.

Circle Geometry   **Name:** ………………………

30.

The diagram shows two circles intersecting at K and M.

From point A and B on the arc of the larger circle, lines are drawn through M to meet the smaller circle at P and Q respectively.

The lines AB and QP meet at O.

Answer on the insert provided.

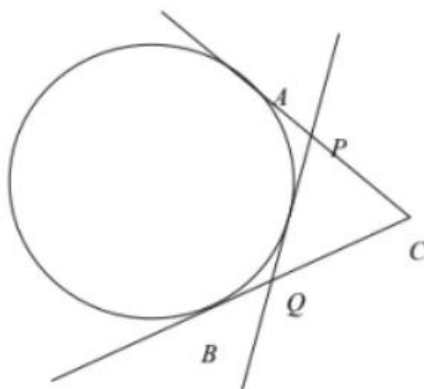i.   If $\theta = \angle KAB$ give a reason why $\angle KMQ = \theta$.

ii.   Prove that AKPO is a cyclic quadrilateral.

iii.   Let $\alpha = \angle AKM$. Show that if OBMP is a cyclic quadrilateral, then the points A, K and Q are collinear.

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Circle Geometry   **Name: …………………………**

31.

A and B are two points on a circle. Tangents at A and B meet at C.0 A third tangent cuts
CA and CB in P and Q respectively, as shown in the diagram.
Show that the perimeter of ΔCPQ is constant and independent of PQ.

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32.   In the diagram, A, B and C are three points on a circle.
        P is another point on the circle, lyimg on the minor arc BC.
        Point L, M and N are the feet of the perpendicular from P to the sides , BC, CA and
        AB respectively.
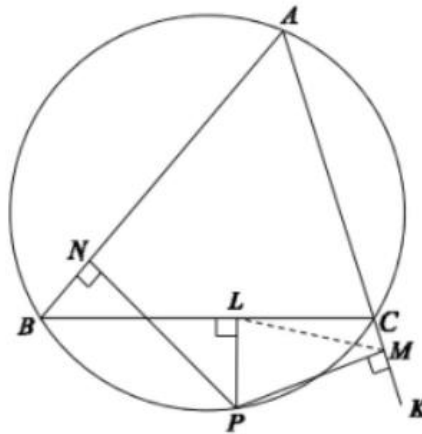
i.   Explain why P, L, N and B are concyclic.

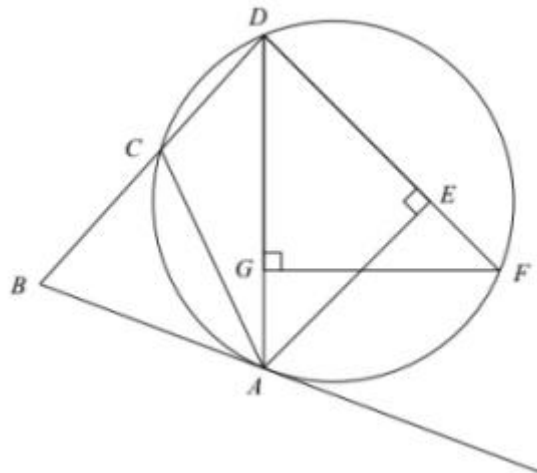ii.   Explain why P, L, C and M are concyclic.

Circle Geometry   **Name: …………………………**

Let $\angle PLM = \alpha$,

iii.   Show that $\angle ABP = \alpha$.

iv.   Hence show that M, L and N are collinear.

Circle Geometry   **Name: …………………………**

33.

In the diagram given, BA is atangent to the circle A at and the secant BD cuts the circle at
      C. DA and DF are two chords such that FG and AE are perpendicular to DA and DF
      respectively.
      copy the diagram.

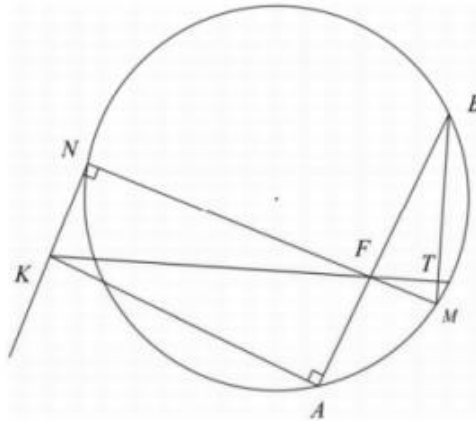i.   Prove that $\angle ACB = \angle BAD$.

Circle Geometry   Name: …………………………

ii.   Explain why AGEF is a cyclic quadrilateral with diameter AF.

iii.   Prove that $\angle AGE = \angle ACD$.

Circle Geometry   **Name: …………………………**

34.

A circle has two chords AB and MN intersecting at F. perpendiculars are drawn to these
     chords at A and at N intersecting at K. KF produced meets MB at T.

i.   Copy or trace the diagram into your writing booklet.

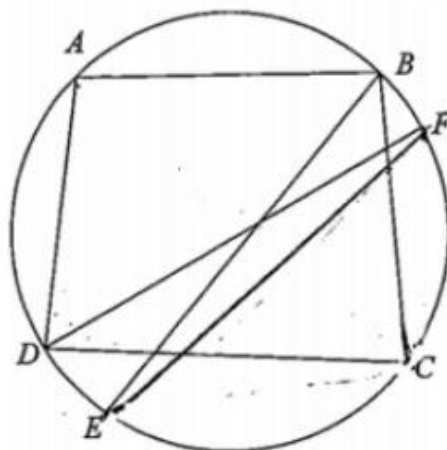**Circle Geometry**   **Name: …………………………**

ii.   Explain why AKNF is a cyclic quadrilateral.

iii.   Prove that KT is perpendicular to MB.

Circle Geometry   **Name: …………………………**

35.   ABCD is a cyclic quadrilateral. Chords BE and DF bisect ∠ABC and ∠ADC respectively.

Copy the diagram and prove that *EF* is a diameter of the circle.