CAPRA COACHING

HSC Extension 1 Mathematics Exam Booklet:

3u Preliminary Content Worksheet

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Easy

1. Find the point dividing the interval form (-3,4) to (5,-2) in the ratio 1:3.

2. Find the acute angle between the lines $y = 3x - 2$ and $x + 2y - 3 = 0$.
Give the answer to the nearest degree.

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3. Find the acute angle between the line $2x + y = 4$ and $x - y = 2$ to the nearest degree.

4. Solve $\frac{x+4}{x-2} \geq 3$.

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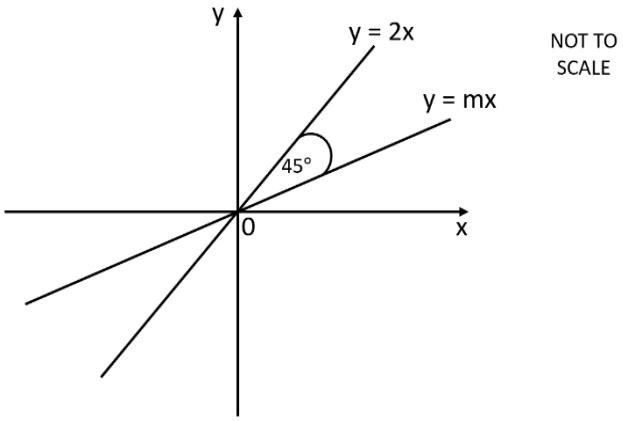
5. The angle between the lines y = mx and y = 2x is 45°, where m > 0, as shown in the diagram below.



Find the value of m.

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) 3

6. A table of values is made to help the curve of y = f(x) is shown below.

x	0	2	4	6	8
f (x)	7	9	14	4	-3

Given that f(x) is continuous over the domain $0 \leq x \leq 8$, the use of Simpson's Rule five

ordinates to estimate $\int_0^8 f(x)\ dx$ will give the result:

A) 28 B) 40

C) 56 D) 80

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7. Solve the inequality $x^2 - 3x < 4$.

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Medium

8. Let A (1, -2) and B (3, 5) be points in the plane. Find the coordinates of the point C which divides the interval AB externally in the ratio 3: 1. ¤

9. Use the substitution u = x – 8 to find $\int_8^{8.5} \frac{dx}{\sqrt{(7-x)(x-9)}}$

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10. Find the distance between the parallel lines $4x + 3y = 12$ and $4x + 3y = 5$.

11. Solve $\frac{2t}{1-t} \geq t$.

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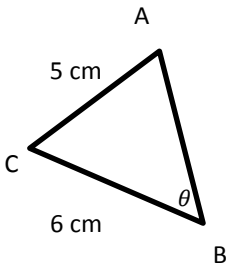
12. For the right triangle ABC



Show that

$$\frac{\sqrt{11}}{6} < \cos\theta < 1$$

13. The equation $3\sin x = \ln x$ has a number of positive solutions, with the smallest solution being close to x = 3.

Use ONE application of Newton's Method to find another approximation to the smallest positive solution. Give this approximation correct to TWO decimal places.

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14. Using the substitution $u = e^x, find \int \frac{e^x}{1+e^{2x}}\ dx$.

15. Using the substitution $u = 9 - x^2, find \int_0^1 6x\sqrt{9 - x^2}\ dx$.

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16. Use the Trapezoidal Rule with 5 function values to obtain an estimate for:

$$\int_{-2}^{6} \log_e \sqrt{x+3} \ \ dx$$

Simplify your answer as much as possible.

17. The graphs of $y = x$ and $y = x^3$ intersect at x = 1. Find the size of the acute angle between these curves at x = 1.

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18. The interval AB has endpoints A (-2, 3) and B (10, 11). Find the coordinates of the point P which divides the interval AB in the ratio 3:1. ¤

19. Find $\int \frac{e^{3\sqrt{x}}}{\sqrt{x}}\ dx$ using the substitution $u = \sqrt{x}$.

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20. (i) Show the function

$$f\,(x) = \log_e x - \sin x + 1$$

has a zero between 0.7 and 0. 8.

(ii) Hence use halving – the – interval method to find the value of this zero, correct to one decimal place.

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21. Let $f\ (x) = \ x^3 + 5x^2 + 17x - 10$. To the equation f(x) = 0 has only one root.

i. Show that the root lies between 0 and 2.

ii. Use one application of the halving the interval method to find a smaller interval containing the root.

iii. Which end of the smaller interval found in part ii is closer to the root? Briefly justify your answer. ¤

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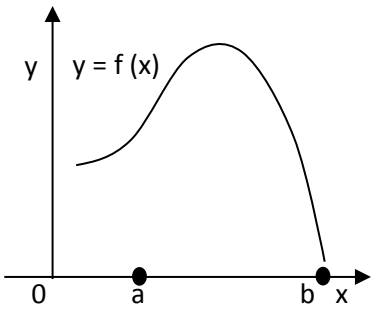
22.



Consider the above graph of y = f(x). The value a shown on the axis is taken as the first approximation to the solution b of f (x) = 0. Is the second approximation obtained by Newton's method a better approximation to b than a is? Give a reason for your answer. ¤

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23. The polynomial equation $f(x) = 8x^3 + 12x^2 - 18x - 20 = 0$ has a root at x = -2.

a. Find all roots of f(x) = 0.

b. Draw a sketch of the graph y = f(x) showing the coordinates of its points of intersection with the axes and all stationary points.

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c. Apply Newton's method once to approximate a root of f(x) = 0 beginning with an initial approximation $x_1 = 1$.

d. Willy chose an initial approximation of $x_1 = 0.49$ and used Newton's method a number of times in order to approximate a root f(x) = 0. State, giving reasons, the root f(x) = 0 to which Willy's approximations are getting closer. (It is not necessary to do additional calculations.) ¤

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24. By using the substitution x= sin t , or otherwise, evaluate $\int_0^{\frac{1}{2}} \sqrt{1 - x^2}\ dx$.

25. i. Draw a sketch of $y = sin^{-1} x$. State the domain and range.

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ii. A region R is bounded by the curve $= sin^{-1} x$, the x – axis and the line x = 1. Use Simpson's Rule with three function values to find an approximation for the area of R. Give your answer correct to 2 decimal places. ¤

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26. a. Find an equation of the normal n to the curve $y = x^4 + 4x^{\frac{3}{2}}$ at the point A (1,5).

b. Find, to the nearest degree, the size of the acute angle between the line n and the line L with equation 2x + 3y – 7 = 0. ¤

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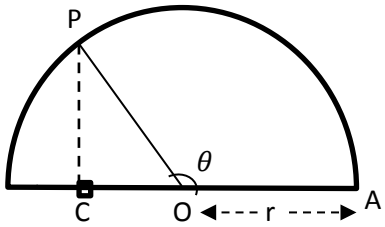
Hard

27.



The point P lies on the circumference of a semicircle of radius r and diameter AB, as shown. The point C lies on AB and PC is perpendicular to AB.

The arc AP subtends an angle θ at the center O, and the length of the arc AP is twice the length of the arc AP is twice the length of PC.

i. Show that $2 \sin \theta = \theta$.

ii. Taking $\theta = 0.18$ as an approximation for the solution to the equation $2\sin\theta = \theta$ between $\frac{\pi}{2}$ and π, use one application of Newton's method to give a better approximation. ¤

28. Find the volume of the solid formed when the region bounded by the x – axis and the curve $y = x\ (8 - x^3)^4$ between x = 0 and x = 2 is rotated about the x – axis. (You may need to use the substitution $u = 8 - x^3$ to evaluate the integral involved.) ¤

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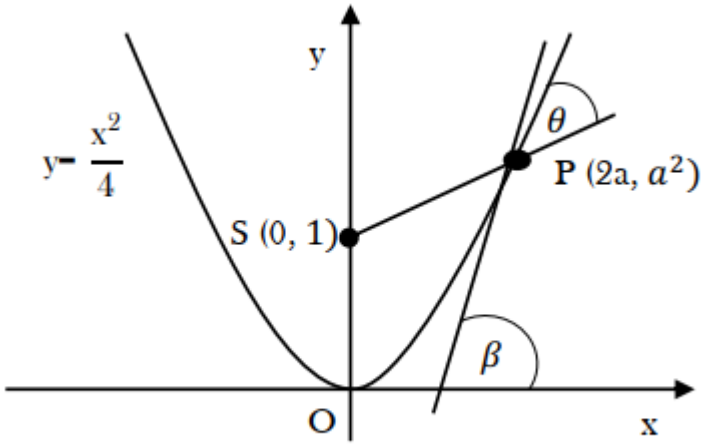
29.



Let P(2a, a^2) be a point on the parabola y= $\frac{x^2}{4}$, and let S be the point (0,1). The tangent to parabola at P makes an angle of β with the x – axis. The angle between SP and the tangent is θ. Assume that a > 0, as indicated.

i. Show that $\tan \beta = a$.

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ii. Show that the gradient of SP is $\frac{1}{2}\left(a - \frac{1}{a}\right)$.

iii. Show that $\tan\theta = \frac{1}{a}$.

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iv. Hence find the value of $\theta + \beta$.

v. Find the solutions of P if $\theta = \beta$. ¤

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30. i. On the same diagram, sketch the graphs of y = x and y =$|2x-1|$.

ii. By using (i) or otherwise, determine for what values of c the equation $|2x-1| = x + c$ has exactly two solutions. ¤

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31. i. Prove that the graph of $y = \ln x$ is concave down for all $x > 0$.

ii. Sketch the graph of $y = \ln x$.

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iii. Suppose 0 < a < b and consider the points A (a, ln a) and B (b, ln b) on the graph of y = ln x.

Find the coordinates of the point P that divides the line segment AB in the ratio 2:1.

iv. By using (ii) and (iii) deduce that $\frac{1}{3}\ln a + \frac{2}{3}\ln b < \ln\left(\frac{1}{3}a + \frac{2}{3}\ln b\right)$. ¤

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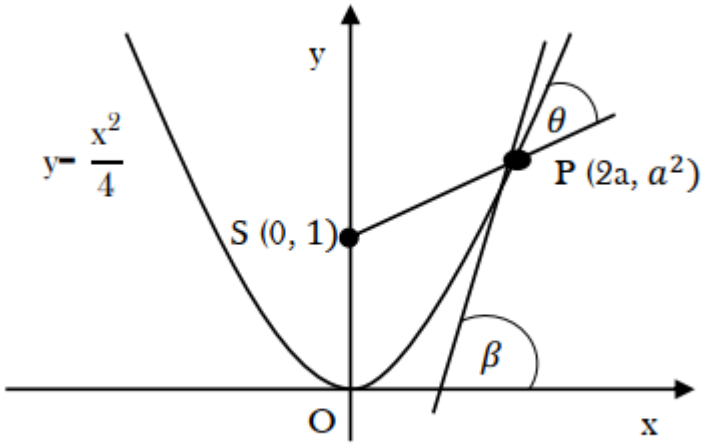
32.



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